

**SECOND YEAR HIGHER SECONDARY MODEL
EXAMINATION, FEBRUARY 2020**

Part – III

Time : 2½ Hours

MATHEMATICS (SCIENCE) Cool-off time : 15 Minutes

Maximum : 80 Scores

General Instructions to Candidates :

- There is a ‘Cool-off time’ of 15 minutes in addition to the writing time.
- Use the ‘Cool-off time’ to get familiar with questions and to plan your answers.
- Read questions carefully before answering.
- Read the instructions carefully.
- Calculations, figures and graphs should be shown in the answer sheet itself.
- Malayalam version of the questions is also provided.
- Give equations wherever necessary.
- Electronic devices except non-programmable calculators are not allowed in the Examination Hall.

വിദ്യാർത്ഥികൾക്കുള്ള പൊതുനിർദ്ദേശങ്ങൾ :

- നിർദ്ദിഷ്ട സമയത്തിന് പുറമെ 15 മിനിറ്റ് ‘കൂൾ ഓഫ് ടൈം’ ഉണ്ടായിരിക്കും.
- ‘കൂൾ ഓഫ് ടൈം’ ചോദ്യങ്ങൾ പരിചയപ്പെടാനും ഉത്തരങ്ങൾ ആസൂത്രണം ചെയ്യാനും ഉപയോഗിക്കുക.
- ഉത്തരങ്ങൾ എഴുതുന്നതിന് മുമ്പ് ചോദ്യങ്ങൾ ശ്രദ്ധാപൂർവ്വം വായിക്കണം.
- നിർദ്ദേശങ്ങൾ മുഴുവനും ശ്രദ്ധാപൂർവ്വം വായിക്കണം.
- കണക്ക് കൂട്ടലുകൾ, ചിത്രങ്ങൾ, ഗ്രാഫുകൾ, എന്നിവ ഉത്തരപേപ്പറിൽ തന്നെ ഉണ്ടായിരിക്കണം.
- ചോദ്യങ്ങൾ മലയാളത്തിലും നൽകിയിട്ടുണ്ട്.
- ആവശ്യമുള്ള സ്ഥലത്ത് സമവാക്യങ്ങൾ കൊടുക്കണം.
- പ്രോഗ്രാമുകൾ ചെയ്യാനാകാത്ത കാൽക്കുലേറ്ററുകൾ ഒഴികെയുള്ള ഒരു ഇലക്ട്രോണിക് ഉപകരണവും പരീക്ഷാഘോഷത്തിൽ ഉപയോഗിക്കുവാൻ പാടില്ല.

Answer any 6 questions from 1 to 8. Each carries 3 scores.

(6 × 3 = 18)

1. Let $A = \begin{bmatrix} 1 & 2 & 3 \\ 3 & 1 & 2 \end{bmatrix}$ and $B = \begin{bmatrix} -3 & 1 & 0 \\ 1 & 3 & -2 \end{bmatrix}$

(i) Which of the following is the order of the matrix $A + B$? (1)

(a) 2×2 (b) 3×2

(c) 2×3 (d) 3×3

(ii) Find $3A$ (1)

(iii) Evaluate $3A - B$ (1)



2. (i) If $y = \sin^{-1} x$, find $\frac{dy}{dx}$. (1)

(ii) Hence show that $(1 - x^2) \frac{d^2y}{dx^2} - x \frac{dy}{dx} = 0$. (2)

3. (i) Which of the following is the solution of the differential equation $\frac{dy}{dx} + \sin x = 0$? (1)

(a) $y = C \cos x$ (b) $y = \cos x + C$

(c) $y = \sin x + C$ (d) $y = C \sin x$

(ii) Form the differential equation representing the family of curves $y = a \sin(x + b)$, where a and b are arbitrary constants. (2)

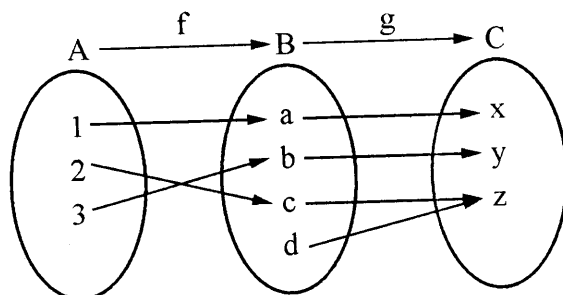
4. Using properties of determinants prove that $\begin{vmatrix} b+c & a & a \\ b & c+a & b \\ c & c & a+b \end{vmatrix} = 4abc$. (3)

5. (i) Find the maximum and minimum values of f , if any of the function $f(x) = |x| + 3$, $x \in \mathbb{R}$. (2)

(ii) What is the absolute maximum value of the function $f(x) = |x| + 3$, $x \in [-3, 2]$? (1)

6. (i) Write a function which is not continuous at $x = 0$ and justify your answer. (2)
- (ii) Check the continuity of the function $f(x) = \begin{cases} x + 2, & \text{if } x < 0 \\ -x + 2, & \text{if } x > 0 \end{cases}$ (1)

7. Consider the arrow diagram of the functions f and g .



- (i) Check whether the functions f and g are bijective. Justify. (1)
- (ii) Write the function $g \circ f$. (1)
- (iii) Is $g \circ f$ a bijective function? Justify. (1)
8. (i) If α, β, γ are the direction angles of a vector, then which the following can be $\alpha + \beta$? (1)
- (a) 80° (b) 60°
- (c) 120° (d) Can't be determined.
- (ii) Find a direction cosines of the line passing through the points $(2, 8, 3)$ and $(4, 5, 9)$. (2)

Answer any 8 questions from 9 to 18. Each carries 4 scores.

(8 × 4 = 32)

9. (i) If $\tan^{-1} x = \frac{\pi}{10}$, then the value of $\cot^{-1} x$ is (1)
- (a) $\frac{\pi}{5}$ (b) $\frac{2\pi}{5}$
- (c) $\frac{3\pi}{5}$ (d) $\frac{4\pi}{5}$
- (ii) Find the value of $\sin\left(2 \tan^{-1} \frac{2}{3}\right) + \cos\left(\tan^{-1} \sqrt{3}\right)$. (3)

10. Let $A = \{-1, 0, 1\}$

- (i) Give reason why the operation defined by $a \otimes b = \frac{a}{b}$ is not a binary operation on A . (1)
- (ii) (a) Write a binary operation $*$ on A . (1)
- (b) Find $(-1 * -1) * -1$. (1)
- (iii) How many binary operations are possible on A ? (1)

11. (i) Find the equation of a line L passing through the points $(-1, 0, 2)$ and $(2, 1, 3)$. (2)
- (ii) If $\vec{c} = \hat{i} + \hat{j} + \lambda\hat{k}$ be a vector perpendicular to the above line, then find λ . (1)
- (iii) Find the equation of a plane on which the line L lies. (1)

12. Find $\frac{dy}{dx}$ of the following :

- (i) $y = \sec(\tan x)$ (1)
- (ii) $x^y = y^x$ (3)

13. (i) Find $\int \tan^{-1} x \, dx$. (2)
- (ii) Hence find the area bounded by the curve $y = \tan^{-1} x$ with X - axis from $x = 0$ and $x = 1$. (2)

14. Solve the differential equation $\frac{dy}{dx} = \frac{x^2 + y^2}{2xy}$. (4)



15. Consider a plane which is equidistant from the points $(1, 2, 1)$ and $(3, 4, 7)$.

- (i) Which of the following is a point on the plane ? (1)
- (a) $(1, 3, 1)$ (b) $(4, 2, 3)$
- (c) $(2, 3, 4)$ (d) $(1, 3, 6)$
- (ii) Find the equation of the above plane. (3)

16. Let $\vec{a} = 2\hat{i} + \hat{j} - 3\hat{k}$ and $\vec{b} = 4\hat{i} + \hat{j} + \hat{k}$ be two vectors.

- (i) Find a vector \vec{c} perpendicular to \vec{a} and \vec{b} . (1)
- (ii) Find the volume of the parallelepiped with co-initial vectors \vec{a} , \vec{b} and \vec{c} . (1)
- (iii) If \vec{c} is rotated in such a way that it makes 60° with its initial direction, then what is the volume of the new parallelepiped formed? (2)

17. (i) Evaluate $\int_0^\pi x \sin x \, dx$. (2)

- (ii) Hence evaluate the area bounded by the curve $y = x \sin x$ between $x = -\pi$ and $x = \pi$. (2)

18. In a ladies hostel, 60% of the students read Hindi newspaper, 40% read English newspaper and 20% read both Hindi and English newspapers. A student is selected at random.

- (i) Find the probability that she reads neither Hindi nor English newspapers. (2)
- (ii) If she reads Hindi newspaper, find the probability that she reads English newspaper too. (1)
- (iii) If she reads English newspaper, find the probability that she reads Hindi newspaper too. (1)



Answer any 5 questions from 19 to 25. Each carries 6 scores.

(5 × 6 = 30)

19. (i) Construct a 3×3 matrix A, where elements are given by $a_{ij} = 2i - j$. (2)
- (ii) Verify that $C = A - A'$ is a skew symmetric matrix. (2)
- (iii) Verify that C^2 is a symmetric matrix. (2)

20. Consider the matrix $A = \begin{bmatrix} 1 & 3 & 6 \\ -1 & -1 & 2 \\ 1 & 1 & 5 \end{bmatrix}$

- (i) Find $|A|$. (1)
- (ii) Verify that $A \times \text{adj}A = |A|I$ (4)
- (iii) Evaluate $|A^{-1}|$. (1)

21. Integrate the following with respect to x :
- $\frac{1}{x^2 - 6x + 13}$ (3)
 - $\frac{\cos x}{(\sin x - 1)(\sin x - 2)}$ (3)
22. Let a pair of dice be thrown and the random variable X be the sum of the numbers that appear on the two dice.
- Write the probability distribution of X . (3)
 - Find variance of X . (3)
23. (i) Consider the function $f(x) = 2x^3 - 6x^2 + 1$.
- Find the equation of tangents parallel to X -axis. (2)
 - Find the intervals in which the function f is decreasing. (2)
- (ii) The length x of a rectangle is decreasing at the rate of 5 cm/s and the width y is increasing at the rate of 4 cm/s. When $x = 8$ cm and $y = 6$ cm, find the rate of change of the area of the rectangle. (2)
24. Let $\vec{a} = 2\hat{i} - 4\hat{j} + 5\hat{k}$ and $\vec{b} = \hat{i} - 2\hat{j} - 8\hat{k}$ be two vectors.
- Find a vector \vec{c} representing a diagonal of the parallelogram with \vec{a} and \vec{b} as the adjacent sides. (2)
 - Find the projection of \vec{b} on \vec{c} . (2)
 - Find the angle between the vectors \vec{c} and \vec{a} . (2)
25. Consider the following L.P.P.
 Maxmise : $Z = 600x + 400y$
 Subject to the constraints :
- $$x + 2y \leq 12$$
- $$2x + y \leq 12$$
- $$4x + 5y \geq 20$$
- $$x \geq 0, y \geq 0$$
- Draw the feasible region. (3)
 - Solve the LPP. (3)

